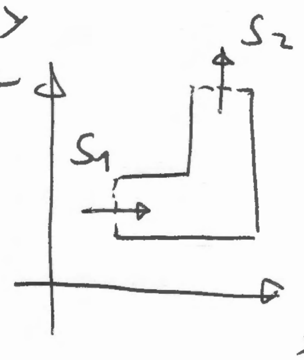
1) eq. bilancio della massa

$$\frac{d}{dt}\int_{V(t)} \rho \, dV + \oint_{\partial V(t)} \rho (u_j - V_j) n_j \, dS = 0$$



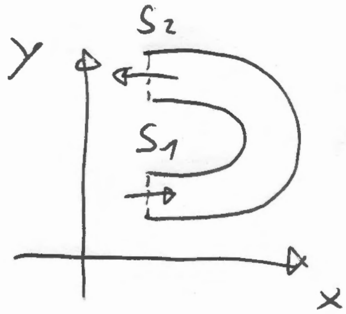
a) $V = 0$, volume controllo fisso, flusso staz. incomprimibile

$$\int_{S_1} \rho \, u_j|_{S_1} n_j \, dS + \int_{S_2} \rho \, u_j|_{S_2} n_j \, dS = 0$$

S_1: $\hat{n} = (-1, 0, 0)$ $u = (U_\infty, 0, 0)$

S_2: $\hat{n} = (0, 1, 0)$ $u = (0, u_2|_{S_2}, 0)$

$$-\rho U_\infty S_1 + \rho \, u_2|_{S_2} S_2 = 0$$

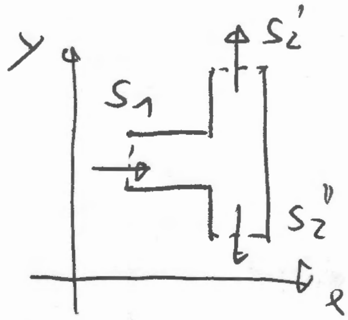


b) $$\int_{S_1} \rho \, u_j|_{S_1} n_j \, dS + \int_{S_2} \rho \, u_j|_{S_2} n_j \, dS = 0$$

S_1: $\hat{n} = (-1, 0, 0)$ $u = (U_\infty, 0, 0)$

S_2: $\hat{n} = (-1, 0, 0)$ $u = (u_1|_{S_2}, 0, 0)$

$$-\rho U_\infty S_1 - \rho \, u_1|_{S_2} S_2 = 0$$



c) $$\int_{S_1} \rho \, u_j|_{S_1} n_j \, dS + \int_{S_2'} \rho \, u_j|_{S_2'} n_j \, dS + \int_{S_2''} \rho \, u_j|_{S_2''} n_j \, dS = 0$$

S_1: $\hat{n} = (-1, 0, 0)$ $u = (U_\infty, 0, 0)$

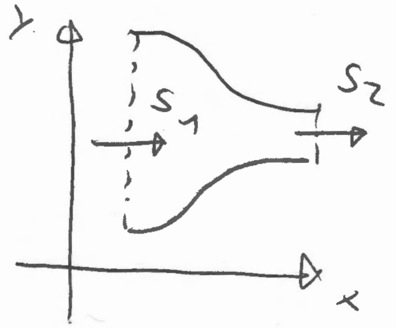
S_2': $\hat{n} = (0, 1, 0)$ $u = (0, u_2|_{S_2'}, 0)$

S_2'': $\hat{n} = (0, -1, 0)$ $u = (0, u_2|_{S_2''}, 0)$

$$u_2|_{S_2''} = -u_2|_{S_2'}$$

$$-\rho U_\infty S_1 + \rho u_2\big|_{S_2'} S_2 - \rho u_2\big|_{S_2''} S_2 = 0$$

$$-\rho U_\infty S_1 + 2\rho u_2\big|_{S_2'} S_2 = 0$$



d) $$\int_{S_1} \rho u_j\big|_{S_1} n_j \, ds + \int_{S_2} \rho u_j\big|_{S_2} n_j \, ds = 0$$

S_1: $\hat{n} = (-1, 0, 0)$ $\quad u = (U_\infty, 0, 0)$

S_2: $\hat{n} = (1, 0, 0)$ $\quad u = (u_1\big|_{S_2}, 0, 0)$

$$-\rho U_\infty S_1 + \rho u_1\big|_{S_2} S_2 = 0$$

2) eq. bilancio di massa

$$\frac{d}{dt}\int_{V(t)} \rho u_i \, dV + \oint_{\partial V(t)} \rho u_i (u_j - v_j) n_j \, ds = \int_{V(t)} \rho f_i \, dV + \oint_{\partial V(t)} t_i \, ds$$

$$t_i = -p n_i + \sigma_{ij}$$

stazionario, incomprimibile, v. controllo fisso, no f. massa, viscose $\Rightarrow t_i = -p n_i$

a) $$\int_{S_1} \rho u_i\big|_{S_1} u_j\big|_{S_1} n_j \, ds + \int_{S_2} \rho u_i\big|_{S_2} u_j\big|_{S_2} n_j \, ds = \int_{S_1 \cup S_2} t_i \, ds + \int_{\partial B} t_i \, ds$$

~~sopp~~ $i = 1$)

$$-\int_{S_1} \rho U_\infty^2 \, ds = -\int_{S_1} p\big|_{S_1} n_1 \, ds - \int p\big|_{S_2} n_1 \, ds - F_{B_1}$$

$$-\rho U_\infty^2 S_1 = p_\infty S_1 - F_{B_1} \Rightarrow F_{B_1} = p_\infty S_1 + \rho U_\infty^2 S_1$$

$i=2)$ $\int_{S_2} \rho v_2|_{S_2}^2 ds = -\int_{S_1} p|_{S_1} n_2 ds - \int_{S_2} p|_{S_2} n_2 ds = F_{B_2}$

$\rho v_2|_{S_2}^2 S_2 = -F_{B_2}$

b) $\int_{S_1} \rho v_i|_{S_1} v_j|_{S_1} n_j ds + \int_{S_2} \rho v_i|_{S_2} v_j|_{S_2} n_j ds = \int_{S_1 \cup S_2} t_i ds - F_{B_i}$

$S_1: \ p = p_\infty \quad S_2: p = 0$

$i=1)$ $-\int_{S_1} \rho V_\infty^2 ds - \int_{S_2} \rho v_1|_{S_2}^2 ds = \int_{S_1} p_\infty n_1 ds - \int_{S_2} p n_1 ds - F_{B_1}$

$-\rho V_\infty^2 S_1 - \rho v_1|_{S_2}^2 S_2 = p_\infty S_1 - F_{B_1}$

$i=2)$ $0+0 = F_{B_2}$

c) $\int_{S_1} \rho v_i|_{S_1} v_j|_{S_1} n_j ds + \int_{S_2'} \rho v_i|_{S_2'} v_j|_{S_2'} n_j ds + \int_{S_2''} \rho v_i|_{S_2''} v_j|_{S_2''} n_j ds = \int_{S_1 \cup S_2' \cup S_2''} t_i ds - F_{B_i}$

$S_1 : p = p_\infty \quad S_2', S_2'' : p = 0$

$i=1)$

$\int_{S_1} \rho \bar{V}_\infty^2 ds = -\int_{S_1} p n_1 ds - \int_{S_2'} p n_1 ds - \int_{S_2''} p n_1 ds - F_{B_1}$

$-\rho V_\infty^2 S_1 = p_\infty S_1 - F_{B_1}$

$i=2)$

$$\int_{S_2'} \rho\, u_2\big|_{S_2'}^2 \, ds - \int_{S_2''} \rho\, u_2\big|_{S_2''}^2 \, ds = -F_{B_2}$$

$$u_2\big|_{S_2''} = -u_2\big|_{S_2'} \Rightarrow \rho\, u_2\big|_{S_2'}^2 S_2 - \rho\, u_2\big|_{S_2'}^2 S_2 = 0 = -F_{B_2}$$

d) $$\int_{S_1} \rho\, u_i\big|_{S_1} u_j\big|_{S_1} n_j \, ds + \int_{S_2} \rho\, u_i\big|_{S_2} u_j\big|_{S_2} n_j \, ds = \int_{S_1 \cup S_2} t_i \, ds - F_{Bi}$$

$S_1 : p = p_\infty \quad S_2 : p = 0$

$i=1)$

$$-\int_{S_1} \rho\, U_\infty^2 \, ds + \int_{S_2} \rho\, u_1\big|_{S_2}^2 \, ds = -\int_{S_1} p n_1 \, ds - \int_{S_2} p n_1 \, ds - F_{B_1}$$

$$-\rho\, U_\infty^2 S_1 + \rho\, u_1\big|_{S_2}^2 S_2 = p_\infty S_1 - F_{B_1}$$

$i=2)$ $\quad F_{B_2} = 0$

3) $1 \text{ atm} \simeq 101{,}000 \text{ Pa}$

$Q = uS = 12 \text{ m}^3/\text{min} \quad \rho = 1000 \text{ kg/m}^3$

$S_1 : D_1 = 15 \text{ cm} \quad p_1 = 3.58 \text{ atm}$

$S_2 : D_2 = 10 \text{ cm} \quad p_2 = 1 \text{ atm}$

$$F_{B_1} = \rho U_\infty^2 S_1 - \rho u_1|_{S_2}^2 S_2 + p_\infty S_1 - p_2 S_2$$

$$U_\infty = \frac{Q}{S_1} \cong 11.3 \text{ m/s} \qquad u_1|_{S_2} = \frac{Q}{S_2} \simeq 25.5 \text{ m/s}$$

$S_1 \simeq 0.018 \text{ m}^2 \quad S_2 \cong 0.008 \text{ m}^2 \qquad Q = 0.2 \text{ m}^3/\text{s}$

$$F_{B_1} = 2298 - 5202 + 6508 - 808 = 2796 \text{ N}$$

4) $$\int_{S_1} \rho u_j|_{S_1} n_j \, dS + \int_{S_2} \rho u_j|_{S_2} n_j \, dS = 0$$

$S_1 : \hat{n} = (-1, 0, 0) \quad u = (U_\infty, 0, 0)$

in coordinate polari $\quad \hat{n} = (0, -1, 0) \quad u = (0, U_\infty, 0)$

$S_2 : \hat{n} = (1, 0, 0) \quad u = (u_r|_{S_2}, 0, 0)$

$$-\int_{S_1} \rho U_\infty \, dS + \int_{S_2} \rho u_r \, dS = 0 \qquad -\rho U_\infty S_1 + \rho u_r S_2 = 0$$

eq. momento della q.d.m.

$$\frac{\partial}{dt}\int_{V(t)} r\times\rho u\, dV + \oint_{\partial V(t)} r\times\rho u(u-U)\cdot n\, dS = \int_{V(t)} r\times f\, dV +$$

$$+\int_{\partial V(t)} r\times t\, dS$$

$U=0$, v. controllo fisso, flusso staz. incomprimibile, f. di massa e viscose nulle.

$$\int_{S_1\cup S_2} r\times\rho u\, u\cdot\hat{n}\, dS = \int_{S_1\cup S_2} r\times t\, dS - M_B$$

S_1: $r = R_1\hat{e}_r$ $u=(0,U_\infty,0)$ $\hat{n}=(0,-1,0)$

S_2: $r = R_2\hat{e}_r$ $u=(u_1|_{S_2},0,0)$ $\hat{n}=(1,0,0)$

$S_1)$ $u\cdot\hat{n} = U_\infty$

$$r\times u = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_z \\ R_1 & 0 & 0 \\ 0 & U_\infty & 0 \end{vmatrix} = \hat{e}_z R_1 U_\infty$$

$$\int_{S_1} r\times\rho u\, u\cdot\hat{n}\, dS = -\rho R_1 U_\infty^2 S_1 \hat{e}_z$$

$$-\int_{S_1} r\times p_\infty \hat{n}\, dS = p_\infty R_1 S_1 \hat{e}_z$$

$S_2)$ $\quad v \cdot \hat{n} = v_1\big|_{S_2}$

$$r \times v = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_z \\ R_2 & 0 & 0 \\ v_1\big|_{S_2} & 0 & 0 \end{vmatrix} = 0$$

$$\int_{S_2} r \times \rho v\, v \cdot \hat{n}\, ds = 0$$

$$\int_{S_2} r \times p \hat{n}\, ds = 0$$

quindi si ha:

$$\left(-\rho R_1 V_\infty^2 S_1 - p_\infty R_1 S_1\right)\hat{e}_z = -M_B$$