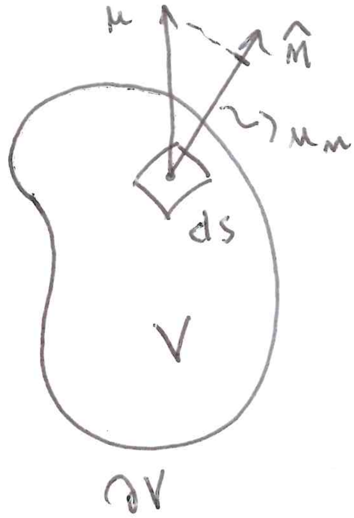
ES 1 TEOREMA DI GAUSS



$$\int_V \nabla \cdot u \, dV = \oint_{\partial V} u \cdot \hat{n} \, dS$$

$$\int_V \frac{\partial u_i}{\partial x_i} dV = \oint_{\partial V} u_i n_i \, dS$$

ES 2

$$\int_V \nabla \cdot T \, dV = \oint_{\partial V} T \cdot \hat{n} \, dS$$

$$\int_V \frac{\partial T_{ij}}{\partial x_j} dV = \oint_{\partial V} T_{ij} n_j \, dS$$

ES 3 TEOREMA DI GREEN

$$\int_V \frac{\partial \phi}{\partial x_i} dV = \oint_{\partial V} \phi n_i \, dS$$

Infatti in notazione vettoriale si scrive

$$\nabla \phi = \nabla \cdot (\phi I) \qquad I \text{ tensore identità}$$

$$(\phi I)_{ij} = \phi \delta_{ij} = \Phi_{ij}$$

$$(\nabla \cdot \Phi)_i = \frac{\partial \Phi_{ij}}{\partial x_j}$$

$$\int_V \frac{\partial \Phi_{ij}}{\partial x_j} dV = \oint_{\partial V} \Phi_{ij} n_j \, dS = \oint_{\partial V} \phi \, \delta_{ij} n_j \, dS$$

$$= \oint_{\partial V} \phi \, n_i \, dS$$

quindi $$\int_V (\nabla p)_i \, dV = \oint_{\partial V} p \, n_i \, dS$$

Invece

$$\int_V \nabla \times u \, dV = \oint_{\partial V} n \times u \, dS$$

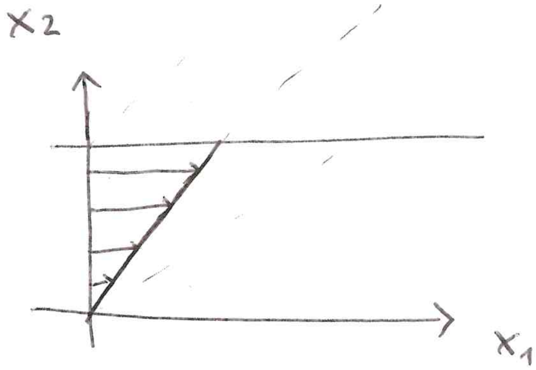
Per componenti infatti risulta

$$\int_V \varepsilon_{ijk} \frac{\partial u_k}{\partial x_j} dV = \oint_{\partial V} \varepsilon_{ijk} u_k n_j \, dS$$

ES 4

$$\oint_{\partial S} a \cdot t \, d\ell = \int_S (\nabla \times e) \cdot n \, dS$$

$$\oint_{\partial S} a_i t_i \, d\ell = \int_S \varepsilon_{ijk} \frac{\partial a_k}{\partial x_j} n_i \, dS$$



ES 5

CAMPO 1 $\quad u = (S x_2, 0, 0)$

$$(\nabla u)_{ij} = \frac{\partial u_i}{\partial x_j}$$

$$E = \mathrm{Sym}\, \nabla u = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\Omega = \mathrm{Skew}\, \nabla u = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

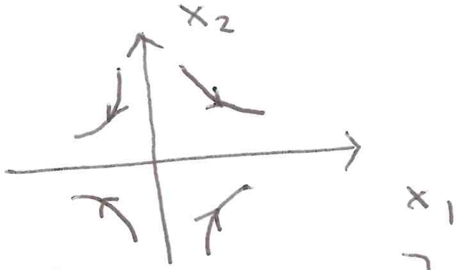
$$\nabla u = \begin{bmatrix} 0 & S & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \nabla u^T = \begin{bmatrix} 0 & 0 & 0 \\ S & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E = \frac{1}{2}\left(\nabla u + \nabla u^T\right) = \begin{bmatrix} 0 & S/2 & 0 \\ S/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Omega = \frac{1}{2}\left(\nabla u - \nabla u^T\right) = \begin{bmatrix} 0 & S/2 & 0 \\ -S/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

CAMPO 2

$$u = (a x_1, -a x_2, 0)$$

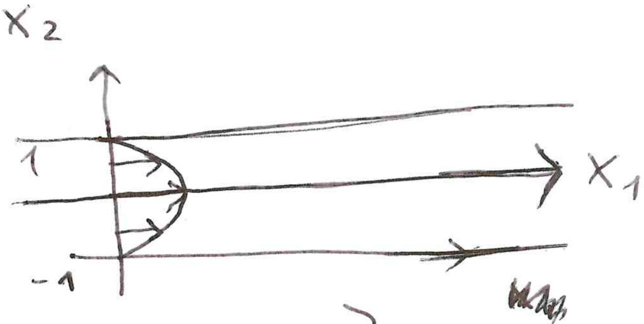


$$\nabla u = \begin{bmatrix} a & -a & 0 \\ 0 & -a & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \nabla u^T = \begin{bmatrix} a & 0 & 0 \\ -a & -a & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a & -a/2 & 0 \\ -a/2 & -a & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0 & -a/2 & 0 \\ a/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

CAMPO 3

$$u = (S(x_2^2 - 1), 0, 0)$$



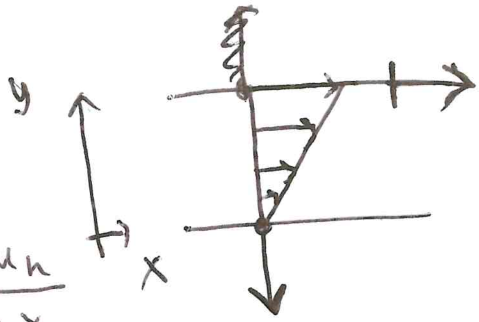
$$u = \begin{bmatrix} 0 & 2Sx_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \nabla u^T = \begin{bmatrix} 0 & 0 & 0 \\ 2Sx_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & Sx_2 & 0 \\ Sx_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0 & Sx_2 & 0 \\ -Sx_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

ES6

$$\Sigma = 2\mu E + \lambda \, T_r E \, I$$

$$\Sigma_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k}$$



$u = (S x_2, 0, 0)$ CAMPO PIANO

$$E = \begin{bmatrix} 0 & S/2 \\ S/2 & 0 \end{bmatrix} \qquad t_r E = 0 \qquad \Sigma = \begin{bmatrix} 0 & \mu S \\ \mu S & 0 \end{bmatrix}$$

$$m_1 = (0, -1) \qquad \Sigma \cdot m_1 = t_{(m_1)} = \begin{bmatrix} 0 & \mu S \\ \mu S & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$= \begin{pmatrix} -\mu S \\ 0 \end{pmatrix}$$

$$m_2 = (1, 0) \qquad \Sigma \cdot m_2 = t_{(m_2)} = \begin{bmatrix} 0 & \mu S \\ \mu S & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ \mu S \end{pmatrix}$$

$u(a x_1, -e x_2, 0)$ CAMPO PIANO

$$E = \begin{bmatrix} a & e/2 \\ ea & -a \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 2\mu a & \mu e \\ \mu e & -2\mu e \end{bmatrix}$$

$$\Sigma \cdot m_1 = \begin{bmatrix} 2\mu e & \mu e \\ \mu e & -2\mu e \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{pmatrix} e \\ 2\mu a \end{pmatrix} \qquad \Sigma \cdot m_2 = \begin{pmatrix} 2\mu e \\ \mu e \end{pmatrix}$$

S7

$$=\left(S(x_2^2-1), 0, 0 \right) \qquad \text{CAMPO PIANO}$$

$$x_1) = A x_1 + B$$

$$= -p I + \Sigma = 2\mu E + (\lambda \, t_2 E - p) I$$

$$= \begin{bmatrix} 0 & S x_2 \\ S x_2 & 0 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 2\mu S x_2 \\ 2\mu S x_2 & 0 \end{bmatrix} - \begin{bmatrix} A x_1 + B & 0 \\ 0 & A x_1 + B \end{bmatrix}$$

$$= \begin{bmatrix} -(A x_1 + B) & 2\mu S x_2 \\ 2\mu S x_2 & -(A x_1 + B) \end{bmatrix}$$

$$m_1) = T \cdot M_1 = \begin{pmatrix} -2\mu S x_2 \\ A x_1 + B \end{pmatrix}$$

$$n_2) = T \cdot M_2 = \begin{pmatrix} -(A x_1 + B) \\ 2\mu S x_2 \end{pmatrix}$$